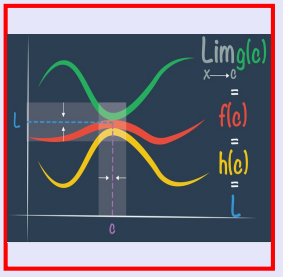
Math 261

Spring 2023

Lecture 15



Feb 19-8:47 AM

Suppose $f(x) = \frac{x+2}{x-3}$

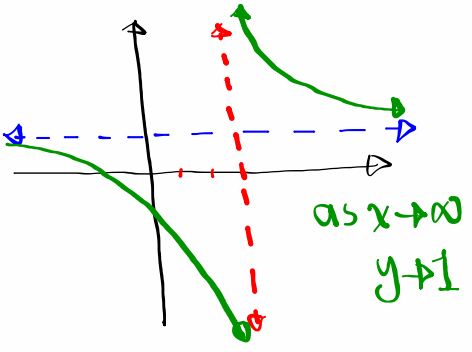
$f(10) \approx 1.714$

$f(100) \approx 1.052$

$f(1000) \approx 1.005$

$f(1{,}000{,}000) \approx 1.000$

work on SG 6

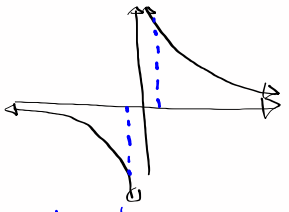


$\lim_{x \to \infty} \frac{x+2}{x-3} = 1$

Mar 1-9:49 AM

limits as $x \to \infty$ or $x \to -\infty$:

$f(x) = \frac{1}{x}$



$\lim_{x\to\infty} \frac{1}{x} = 0$

$\lim_{x\to-\infty} \frac{1}{x} = 0$

$\lim_{x\to0^+} \frac{1}{x} = \infty$

$\lim_{x\to0^-} \frac{1}{x} = -\infty$

$\lim_{x\to\infty} \frac{x+2}{x-3} = \frac{\infty}{\infty}$ I.F.

Divide by highest Power of x

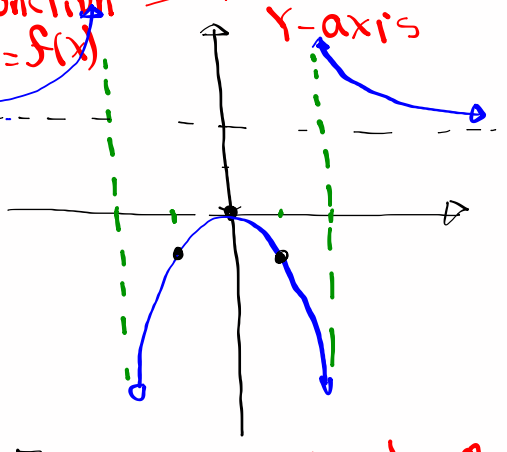
$$\lim_{x\to\infty} \frac{\frac{x+2}{x}}{\frac{x-3}{x}} = \lim_{x\to\infty} \frac{\frac{x}{x}+\frac{2}{x}}{\frac{x}{x}-\frac{3}{x}} = \lim_{x\to\infty} \frac{1+\frac{2}{x}}{1-\frac{3}{x}} \quad (\frac{2}{x}\to 0,\ \frac{3}{x}\to 0)$$

$$= \frac{1+0}{1-0} = \boxed{1}$$

Mar 2-8:53 AM

$\lim_{x\to\infty} \frac{2x^2}{x^2-4}$

even function $f(-x) = f(x)$ → sym. about y-axis



$f(1) = \frac{2}{-3}$

$f(-1) = \frac{2}{-3}$

$\lim_{x\to\infty} \frac{2x^2}{x^2-4} = \frac{\infty}{\infty}$ I.F.

$\lim_{x\to\infty} \frac{1}{x^2} = 0$

$$\lim_{x\to\infty} \frac{\frac{2x^2}{x^2}}{\frac{x^2}{x^2}-\frac{4}{x^2}} = \lim_{x\to\infty} \frac{2}{1-\frac{4}{x^2}} \quad (\frac{4}{x^2}\to 0) = \frac{2}{1} = \boxed{2}$$

Mar 2-8:58 AM

Evaluate $\lim_{x\to\infty} \frac{3x}{\sqrt{25x^2-1}} = \frac{\infty}{\infty}$ I.F.

$25x^2 - 1 \approx 25x^2$

$\sqrt{25x^2-1} \approx \sqrt{25x^2} = 5x$

⇒ Divide everything by x

as $x \to \infty$, $x = \sqrt{x^2}$

$$\lim_{x\to\infty} \frac{3x}{\sqrt{25x^2-1}} = \lim_{x\to\infty} \frac{\frac{3x}{x}}{\frac{\sqrt{25x^2-1}}{x}}$$

$$= \lim_{x\to\infty} \frac{3}{\sqrt{\frac{25x^2-1}{x^2}}} = \lim_{x\to\infty} \frac{3}{\sqrt{\frac{25x^2}{x^2} - \frac{1}{x^2}}} = \lim_{x\to\infty} \frac{3}{\sqrt{25 - \frac{1}{x^2}}}$$

$$= \frac{3}{\sqrt{25-0}} = \boxed{\frac{3}{5}}$$

Mar 2-9:06 AM

Evaluate $\lim_{x\to\infty} \left(\sqrt{x^2+25} - x\right) = \infty - \infty$ I.F.

$\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty - \infty$

$\frac{25}{\infty} \to 0$

$$\lim_{x\to\infty} \frac{\left(\sqrt{x^2+25} - x\right)\left(\sqrt{x^2+25} + x\right)}{\sqrt{x^2+25} + x}$$

$$= \lim_{x\to\infty} \frac{25}{\sqrt{x^2+25} + x}$$

Divide by x, Recall $x = \sqrt{x^2}$

$$= \lim_{x\to\infty} \frac{\frac{25}{x}}{\sqrt{\frac{x^2}{x^2} + \frac{25}{x^2}} + \frac{x}{x}} = \lim_{x\to\infty} \frac{\frac{25}{x} \to 0}{\sqrt{1 + \frac{25}{x^2} \to 0} + 1}$$

$$= \frac{0}{2} = \boxed{0}$$

Mar 2-9:13 AM

Evaluate $\lim_{x\to\infty}\left(\sqrt{x^2+25x}-x\right) = \infty-\infty$ I.F.

$$= \lim_{x\to\infty}\frac{\left(\sqrt{x^2+25x}-x\right)\left(\sqrt{x^2+25x}+x\right)}{\sqrt{x^2+25x}+x}$$

$$= \lim_{x\to\infty}\frac{25x}{\sqrt{x^2+25x}+x} = \frac{\infty}{\infty}$$ I.F.

$$= \lim_{x\to\infty}\frac{\frac{25x}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{25x}{x^2}}+\frac{x}{x}} = \frac{25}{\sqrt{1+0}+1} = \frac{25}{2} = 12.5$$

Suppose $x = 1000$

$$\sqrt{1000^2+25(1000)}-1000 \approx 12.423$$

Now try $x = 10000$

$$\sqrt{10000^2+25(10000)}-10000 \approx 12.492$$

Now try $x = 1000000$

$$\sqrt{1000000^2+25(1000000)}-1000000$$

as $x\to\infty$, $\sqrt{x^2+25x}-x \to 12.5$ ≈ 12.499

Mar 2-9:21 AM

$$\lim_{x\to-\infty}\frac{3x+4}{2x-4} = \frac{-\infty}{-\infty} = \frac{\infty}{\infty}$$ I.F.

Divide by x

$$\lim_{x\to-\infty}\frac{3x+4}{2x-4} = \lim_{x\to-\infty}\frac{\frac{3x}{x}+\frac{4}{x}}{\frac{2x}{x}-\frac{4}{x}} = \lim_{x\to-\infty}\frac{3+\frac{4}{x}}{2-\frac{4}{x}}$$ ($\frac{4}{x}\to 0$, $\frac{4}{x}\to 0$)

$$= \frac{3}{2}$$

Mar 2-9:34 AM

Find $\lim_{x\to-\infty} \frac{2x}{\sqrt{25x^2-1}}$ → $-.4$

Try $x=-100$

$$\frac{2(-100)}{\sqrt{25(-100)^2-1}} = -.400$$

Try $x=-1000$

$$\frac{2(-1000)}{\sqrt{25(-1000)^2-1}} = -.400$$

Divide by x

Since $x\to-\infty$, $x=-\sqrt{x^2}$

$$\lim_{x\to-\infty} \frac{2x}{\sqrt{25x^2-1}} = \lim_{x\to-\infty} \frac{\frac{2x}{x}}{\frac{\sqrt{25x^2-1}}{x}}$$

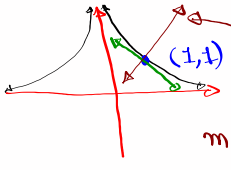
$$= \lim_{x\to-\infty} \frac{2}{\frac{\sqrt{25x^2-1}}{-\sqrt{x^2}}} = \lim_{x\to-\infty} \frac{2}{-\sqrt{\frac{25x^2}{x^2}-\frac{1}{x^2}}}$$ ($\frac{1}{x^2}\to 0$)

$$= \frac{2}{-\sqrt{25}} = \frac{2}{-5} = \boxed{-.4}$$

Mar 2-9:40 AM

Find eqn of the normal line to the graph of $f(x)=\frac{1}{x^2}$ at $x=1$.

even function $f(x)=f(-x)$



Normal line

$m = \frac{-1}{m_{\text{tan. line}}}$

$m = \frac{-1}{f'(1)}$

$$f'(1) = \lim_{h\to0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to0} \frac{\frac{1}{(1+h)^2}-1}{h}$$

$$= \lim_{h\to0} \frac{1-(1+h)^2}{h(1+h)^2} = \lim_{h\to0} \frac{1-1-2h-h^2}{h(1+h)^2}$$

$$= \lim_{h\to0} \frac{h(-2-h)}{h(1+h)^2} = \frac{-2}{1} = -2$$

$m_{\text{Normal line}} = \frac{-1}{-2} = \frac{1}{2}$

$y-1 = \frac{1}{2}(x-1) \Rightarrow \boxed{y=\frac{1}{2}x+\frac{1}{2}}$

Mar 2-9:47 AM